

Dynamical Determination of the Fundamental Couplings

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Abstract

We demonstrate that supergravity models containing the Standard Model, dilaton and modulus naturally lead to dynamical symmetry breaking with excellent phenomenology. We assume primordial supersymmetry breaking in the form of a constant contribution to the superpotential. String inspired relations link fundamental couplings to the dilaton vev. We specialize to a class of models inspired by the 4- D fermionic string. Non-renormalizable terms in the superpotential naturally produce the Higgs mixing parameter μ suitable for our mechanism. We discuss extensions and limitations of our approach.

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A fairly complete picture of physics from the Planck scale to the weak scale can be painted in the framework of string theory. However, although possible mechanisms for supersymmetry breaking and fixing the vevs of moduli fields exist, these issues remain unclear. Perhaps the most complete and popular mechanism relies on gaugino condensation in the hidden sector [1]. This approach has at least two major phenomenological problems: first, the vacuum energy without fine-tuning is $\mathcal{O}(\Lambda^4)$ where Λ is the condensation scale and second, a large ratio for $m_0/m_{1/2}$ is uncomfortable in the light of experimental bounds on the gaugino mass [2] and the hierarchy problem [3].

In contrast, no-scale models have been successful in breaking a flat modulus direction by radiative corrections while giving a vacuum energy of $m_{3/2}^4$ and small ratios for $m_0/m_{1/2}$ [4]. However a no-scale mechanism for fixing the dilaton vev has never been demonstrated. The dilaton is of particular interest since it determines the unified gauge coupling [5]

$$g^{-2}(M_X) = \langle S \rangle. \quad (1)$$

In this paper, we present a mechanism for dynamically determining the vev of the dilaton field via radiative symmetry breaking. Section 1 demonstrates this mechanism using a toy supergravity model. Section 2 considers the mechanism in the light of a specific class of string models inspired by 4D free-fermionic string constructions. Section 3 outlines some additional considerations and questions.

1. A Simple Supergravity Model

The simplest supergravity model which includes the dilaton, S , and modulus field, T , and is capable of reproducing the Supersymmetric Standard Model (SSM) with vacuum energy $\mathcal{O}(m_{3/2}^4)$ is constructed from the Kähler function

$$G = -\ln(S + \bar{S}) - 2\ln(T + \bar{T}) + z_i \bar{z}^i + \ln|W|^2 \quad (2)$$

where the z_i are the chiral fields of the SSM (a bar denotes complex conjugate). Generic string considerations [5] indicate that the gauge kinetic function at tree-level is given by

$$f_{ab} = \delta_{ab} S. \quad (3)$$

In addition, we take the superpotential

$$W = w + \mu H_1 H_2 + \lambda_t H_2 Q_3 U_3^c. \quad (4)$$

where we have included a constant term w coming from tree-level supersymmetry breaking in the string (for instance from a generalized compactification scheme [6]) and a Higgs mixing parameter μ whose origin is considered to be unknown at this point and will be considered later.

The model leads to a one-loop corrected Planck-scale scalar potential in Landau gauge [7]

$$V = \frac{1}{8}(g_y^2 + g_2^2)(|H_1|^2 + |H_2|^2)^2 + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - (B\mu H_1 H_2 + c.c.) + \frac{1}{64\pi^2} \text{Str} \mathcal{M}^4 \left(\ln \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) + \eta m_{3/2}^4 \quad (5)$$

where we regard η and μ at the unification scale M_X as free parameters. The soft supersymmetry breaking parameters coming from the model are simply calculated as

$$A(M_X) = 3m_{3/2}, \quad B(M_X) = \left(2 - (S + \bar{S}) \frac{\partial \ln \mu}{\partial S} \right) m_{3/2}, \quad m_0 = m_{1/2} = m_{3/2} \quad (6)$$

and the gravitino mass $m_{3/2}$ is

$$m_{3/2}^2 = e^{\langle G \rangle} = \frac{\langle |W|^2 \rangle}{\langle (S + \bar{S})(T + \bar{T})^2 \rangle}. \quad (7)$$

The construction of the low-energy scalar potential follows from the standard one-loop RGEs. For the Higgs mass terms [8], we include only the contributions of gauge couplings and the top-Yukawa for simplicity. For the cosmological term [7] and supertrace terms [9] we use analytic forms dependent upon the

gauge coupling contributions of squarks and gluinos. We have taken no thresholds in running the equations to M_z .

In treating μ as of unknown origin, we can consider it to depend only on the dilaton field, and further that its rescaled form is given by

$$\hat{\mu} = g\mu(S). \quad (8)$$

In this case, we can take its first and second derivatives with respect to the dilaton field as two more free parameters. We take the rescaled form of the top-Yukawa at the unification scale as

$$\lambda_t(M_X) = g^2 \quad (9)$$

and hence has no T dependence. Motivation for these choices will be given in Section 2.

The occurrence of the gravitino mass in the parameters (6) implies that the minimization with respect to the Higgs fields results in

$$\langle H_{1,2} \rangle = \tilde{H}_{1,2}(\tilde{\mu})m_{3/2} \quad (10)$$

where \tilde{H} is a dimensionless function of $\tilde{\mu}$ ($= \mu/m_{3/2}$) only. We can determine $\tan\beta$ and $m_{3/2}$ as functions of μ by using the known value of M_w :

$$M_w^2 = g_2^2 \langle H_1 \rangle^2 (1 + \tan^2 \beta) \quad \text{with} \quad \tan \beta = \frac{\langle H_2 \rangle}{\langle H_1 \rangle}. \quad (11)$$

Since the only appearance of the modulus field T is in $m_{3/2}$, minimizing in this field gives the “no-scale condition” of [10]:

$$\frac{\partial V}{\partial m_{3/2}} = 0 \quad (12)$$

which determines a relation between $\eta(M_X)$, μ and $\langle S \rangle$. This result implies that the minimization in S only involves explicit S dependence coming from dilaton dependent boundary conditions at the unification scale:

$$5/3g_1^2 = g_2^2 = g_3^2 = \lambda_t = \frac{1}{\text{Re}S} \quad \text{at } M_X. \quad (13)$$

A choice of $\partial\hat{\mu}/\partial S$ determines a value for B through (6). The condition $\partial V/\partial S = 0$ at a value of the dilaton field which reproduces $\alpha_X = 1/24$ then determines $\hat{\mu}$. Stability of the theory must then be verified for that choice of $\partial\hat{\mu}/\partial S$ since the Hessian involves non-trivial mixing of S and H derivatives. Table 1 indicates a range of B for which consistent results can be obtained. All values of the second derivative $\sim \mathcal{O}(m_{3/2}/M_{\text{pl}}^2)$ preserve stability. The smallness of $\langle \tilde{H}_1 \rangle$ allows an easy solution to the condition for an extremum in S since the derivative of $\hat{\mu}$ appears only in D-terms and Higgs mass terms which are proportional to \tilde{H}_1^4 and \tilde{H}_1^2 respectively, and hence make negligible contributions to the determination of the minimum. The fields $\text{Re}(S)$ and $\text{Re}(T)$ have masses $\mathcal{O}(M_w^2/M_{\text{pl}})$ while $\text{Im}(S)$ and $\text{Im}(T)$ are massless at this level of analysis. These fields could have important cosmological implications [11]. The remarkable phenomenology given by this model is a consequence of string inspired choices of boundary conditions and the tight constraints on the location of the minimum.

2. A Class of Globally Supersymmetric String-Inspired Models

Having illustrated a mechanism for determining a (global) minimum in Higgs, dilaton and moduli fields, we broaden our consideration to a class of globally supersymmetric models inspired by the 4- D fermionic string construction. In so doing, we will be led to a natural mechanism for the creation of the μ term, and will provide motivation for the choice of rescaled μ and λ_t outlined in the previous section.

The spectrum of fields can be assigned to string sets (twisted or untwisted) and sectors (of which there are three in each set). The Kähler potentials and soft SUSY breaking parameters corresponding to the different choices are well enumerated [12] and determine the necessary rescalings for the passage to the low energy theory. Conservation of charge on the string world-sheet determines the allowed couplings between

different sets and sectors, and hence restricts the possible soft terms which can arise [13]. In particular, the analysis of [12] indicates that, whatever the trilinear couplings present in the superpotential:

$$A = m_{3/2}, \quad m_{1/2} = m_{3/2}. \quad (14)$$

The natural generalization of (6) can be written

$$B = \left(B_n - (S + \bar{S}) \frac{\partial \ln \mu}{\partial S} \right) m_{3/2} \quad \text{with} \quad -n \leq B_n \leq n \quad (15)$$

where B_n arises from field differentials of the Kähler potential (and may be T dependent) and string-inspired potentials restrict its possible value. Assignments of fields to sectors and sets give, in general, non-universal values for m_0 , therefore although any particular scalar field will be massless or of mass $m_{3/2}$, we expect that choosing

$$\frac{m_0}{m_{3/2}} = 0, 1 \quad (\text{universal}) \quad (16)$$

will cover the range of possible assignments as far as the results of the minimization go. The choices (14), (15) and (16) define a class of models suitable for the minimization procedure of Section 1, when supplemented with suitable boundary conditions for λ_t and μ .

Field normalization and the rescaling of the superpotential required to emulate a globally supersymmetric theory contribute to an effective rescaling of all couplings present in the superpotential [14]. With a T -independent string-derived Yukawa, the restrictions on allowed couplings ensure that the low-energy Yukawas are also T -independent. Further, treating the origin of the μ -term as a non-renormalizable term in the superpotential of the form [15]

$$W_{nr} = \lambda_n \phi^{n-2} H_1 H_2 \quad \Rightarrow \quad \mu = \lambda_n \langle \phi \rangle^{n-2}. \quad (17)$$

It is clear that the rescaling of μ will depend on assignments of the scalar fields H_1 , H_2 and ϕ . Restricting attention to allowed couplings, we can often arrange for the rescaling not to introduce any factors of T into $\hat{\mu}$ or B_n , and hence we can appeal to this freedom in justifying the choice made in Section 1. We choose

$$\hat{\mu} = g \lambda_n \langle \hat{\phi} \rangle^{n-2} \quad (18)$$

as typical of the possible rescalings. The notation $\langle \phi \rangle$ is purposefully vague since we consider the specification of ϕ to be beyond the scope of this paper.

In Figures 1 and 2, we treat B as an unconstrained parameter coming from a string model and show the variation of the top mass and lightest Higgs mass, of $\tan \beta$ and of the value of $\hat{\mu}$ required to enforce an extremum. We note that since the $m_0 = 0$ and $m_0 = m_{3/2}$ contours in Figures 1 and 2 are almost identical, we expect that models with non-universal boundary conditions for m_0 will give similar results. For all values of B , we obtain

$$m_{3/2} = 1.5 \text{ TeV}, \quad 2.2 \text{ TeV} \quad \text{for} \quad m_0 = 0, \quad m_{3/2} \quad \text{respectively.} \quad (19)$$

At a deeper level of analysis, we treat B_n as the defining parameter coming from a string model, and illustrate in Figure 3 the range of B for which $\partial \hat{\mu} / \partial S$ gives a stable theory. Again, all reasonable values of the second derivative of $\hat{\mu}$ preserve stability.

A further step comes from noting that with a string-derived coupling between n string fields with vertex correlation function C [13,16]

$$\lambda_n = C \frac{\sqrt{2}}{(2\pi)^{n-3}} g^{n-2} \quad (20)$$

we can infer from (18) that

$$\frac{\partial \hat{\mu}}{\partial S} = - \left(\frac{n-1}{2S} \right) \frac{\hat{\mu}}{M_{\text{pl}}} \quad (21)$$

where we have restored mass units. Using equation (15), Figure 3 shows contours of constant n which indicate the allowed form of the non-renormalizable term in W .

3. Conclusions and Speculations

Having presented a simple scheme for the dynamical determination of the vevs of all scalar fields in a realistic no-scale model, we are in a position to speculate on the validity and implications of our results. Having included only the top-Yukawa and gauge couplings in the RGEs restricts the range of validity to $\tan\beta < 8$ [17]. However, since our results fall in this range we can feel confident that the inclusion of other Yukawas will not dramatically affect our results. We expect inclusion of the whole spectrum of particles in the supertrace will quantitatively change the results obtained. However this will not invalidate the procedure since $V \rightarrow 0$ in the weak coupling limit ($S \rightarrow \infty$), and $V \rightarrow \infty$ as α_3 blows up ($S \rightarrow 1.27$) – a result which depends on the squark and gluino contributions only. If $V < 0$ for some $S > 1.27$ as in the examples presented, there will be a minimum. A more serious question is raised by the large value for $m_{3/2}$, for which a one-loop corrected potential may not be sufficient [18]. However recent work [19] may indicate that a scale well over 1 TeV is permissible without having to invoke a large degree of fine-tuning.

Comments in Section 1 regarding the implications of small values of the (dimensionless) Higgs vevs also indicate that the inclusion of moduli dependence in μ or B will not change our procedure for determining a minimum since T dependence in V beyond $m_{3/2}$ will appear in terms multiplied by \tilde{H}^2 . However, this additional dependence will put an upper bound on the vev of T since stability of the theory may be affected by large values of $\langle T \rangle$. Additional moduli dependence in the gauge couplings implied by modular invariance [20] appears at one-loop level and will again be negligible.

We note that in principle, string considerations will predict values for η and μ which must reproduce the correct vev for S . However, since string-derived values for these parameters are uncertain, we choose to take the vev of S as known and to regard the values for the parameters required by our procedure as the values which need to be derived from the string. In this way, we can regard these values of η and μ as tests of consistency rather than predictions.

One may question the validity of trying to determine the vev of the dilaton field using only the electro-weak scalar potential. Indeed, realistic string models have extra $U(1)$ or GUT generators broken at a large scale which will presumably dominate the minimization. In principle, a similar analysis is possible including GUT structure. However, the resulting vacuum energy will naturally be $\mathcal{O}(M_{\text{GUT}}^4)$ unless some mechanism is found to tame it.

The discrepancy between a string unification scale generically $\mathcal{O}(10^{17}$ GeV) and a unification scale $\mathcal{O}(10^{16}$ GeV) extrapolated from low energy couplings in the SSM motivates a string-inspired Standard Model with minimal extra particle content to reconcile these scales [21]. The resulting modification of the RGEs for gauge couplings will alter both the value of $g(M_X)$ and the dependence of the various parameters in the low energy Lagrangian on S . This will probably result in major quantitative differences in the results of our mechanism.

The critical assumption in this procedure has been to break supersymmetry at tree-level with a constant term in the superpotential. However, in the light of comments in Section 3 regarding the origin of a μ -term, it is clear from equation (4) that once a μ -term has been created through a singlet field acquiring a vev, W also acquires a vev dynamically. This provides a mechanism for the dynamical breaking of supersymmetry without the tree-level contribution. It is unclear how to reconcile such a supersymmetry breaking mechanism with string no-go theorems. Moreover, a phenomenological difficulty would be that the inferred vev of the modulus field will be $\mathcal{O}(10^{-12}\text{eV})$ and hence we face a decompactification disaster. However it is possible that an analysis of GUT structure and creation of a GUT analogue of the μ -term could overcome this problem.

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B	$\hat{\mu}$	$\langle H_1 \rangle$	$\tan \beta$	$m_{3/2}$	$m_t(\text{pole})$	m_h	m_H
$2.8m_{3/2}$	$2.7m_{3/2}$	$0.033m_{3/2}$	2.2	2.2 TeV	165 GeV	60 GeV	7.0 TeV
$3.7m_{3/2}$	$3.2m_{3/2}$	$0.043m_{3/2}$	1.6	2.2 TeV	153 GeV	38 GeV	8.7 TeV

Table 1. Range of consistent results for minimization of the model $m_0 = 1$, $A = 3$.

Figure Captions

- Fig. 1. Masses of the Top quark and lightest Higgs for the class of model in Section 2 with B (in units of $m_{3/2}$) as a parameter. Dashed lines indicate $m_0 = 0$, solid lines indicate the case $m_0 = m_{3/2}$.
- Fig. 2. Values of $\tan\beta$ and $\hat{\mu}$ (in units of $m_{3/2}$) at the minimum for the class of model in Section 2 with B (in units of $m_{3/2}$) as a parameter. Dashed lines indicate $m_0 = 0$, solid lines indicate the case $m_0 = m_{3/2}$.
- Fig. 3. Range of stability of a model defined by a value of B_n . Solid diagonal lines indicate the number of fields n in a non-renormalizable term which create the required dilaton dependence in a μ -term. The region between dashed lines has $m_0 = 0$ and between solid lines has $m_0 = m_{3/2}$. The dotted region is excluded by equation (15). B is measured in units of $m_{3/2}$.

Mass (GeV)

m_t

m_h

B

Figure 1.

$\hat{\mu}$

$\tan \beta$

B

Figure 2.

B

B_n

Figure 3.

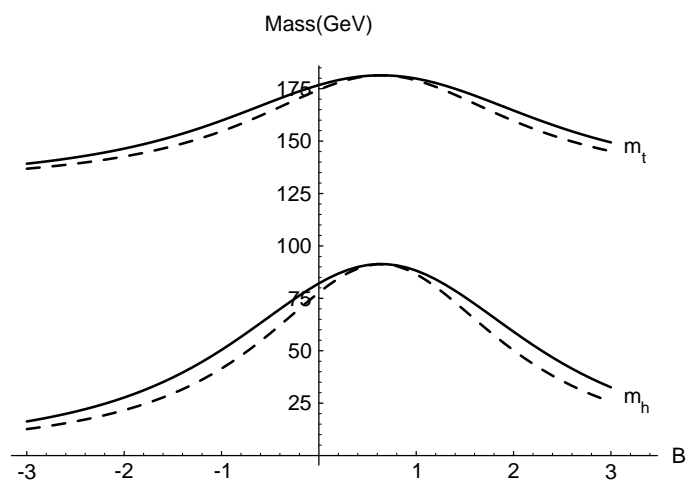


Figure 1.

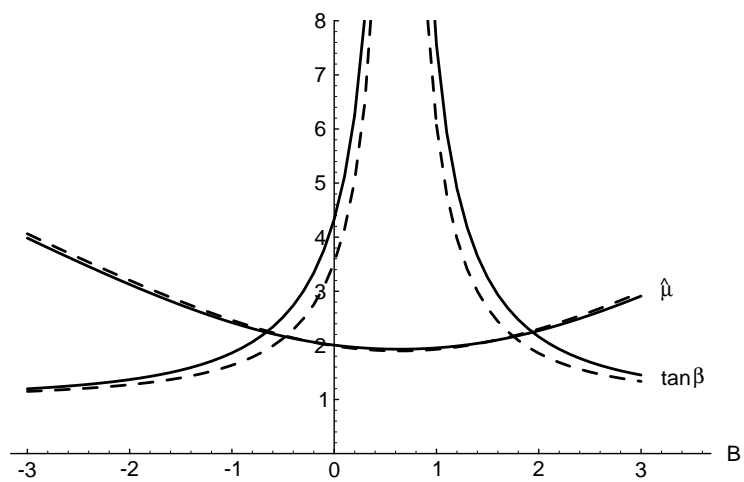


Figure 2.

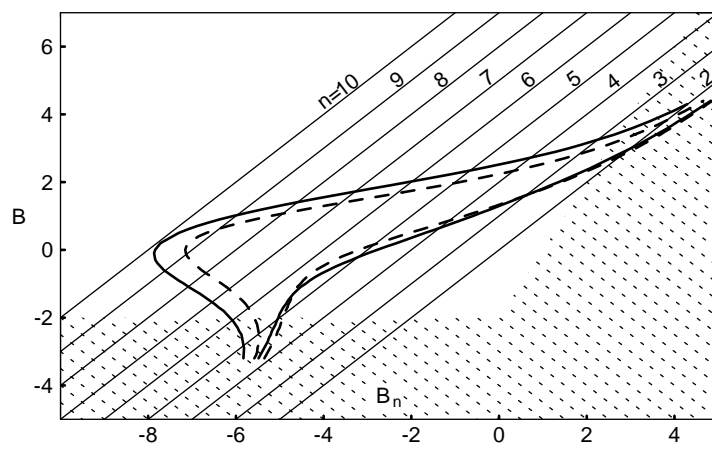


Figure 3.